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# Multi-Objective Differential Evolution (MODE): A New Algorithm for Solving Multi-Objective Optimization Problems

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**ABSTRACT**

*Multi-objective Differential Evolution (MODE), a population based, multi objective search algorithm is proposed by our group earlier. MODE algorithm is applied successfully on industrial problems (Babu et. al., 2005), extensive parametric study of MODE algorithm is also done (Babu and Gujrathi, 2005). In this work, special emphasis is made on the effect of objective space and the decision variable space on the Pareto optimal front. Five different Test Problems are selected for this purpose. Interesting results are obtained which are reported.*

## INTRODUCTION:

Multi-objective Optimization refers to finding one or more feasible solutions, which correspond to extreme values of one or more objectives. The need for finding such optimal solutions in a problem comes mostly from the purpose of either designing a solution for minimum possible cost of fabrication or for maximum possible reliability, or others. Because of such extreme properties of optimal solutions, optimization methods are of great importance in practice, particularly in engineering design, scientific experiments and business decision-making (Deb, 2001). Most of the real world problems involve more than one objective, making multiple conflicting objectives interesting to solve multi-objective optimization problems (MOOP).

Unlike Traditional preference-based methods, Evolutionary Algorithms can find multiple optimal solutions in a single simulation run due to their population-based search algorithms. A detailed account of multi-objective optimization using evolutionary algorithms and some of the applications of Multi-objective Differential Evolution (MODE) algorithms can be found in literatures (Babu et. al., 2005, Babu and Gujrathi, 2005, Deb, 2001, Onwubolu and Babu, 2004).

MODE is an extension of Differential Evolution (DE). DE is found to give better results than Genetic Algorithm (GA) for many optimization problems (Babu and Sastry, 1999; Babu et al., 2005). Earlier, successful application of MODE algorithm is made to carry out the multi-objective optimization of Styrene Reactor (Babu et. al, 2005), parametric study of MODE algorithm on benchmark Test Problems (Babu and Gujrathi, 2005). The schematic of MODE algorithm is shown in Fig. 1. In this work five Test Problems of multi-objective optimization are solved for finding the effect of objective space and decision variable space on Pareto Optimal front using MODE. MODE is tested for the feasible objective space and decision variable space for various known Test Problems. All the Test Problems contain two objective functions and two variables. To solve the algorithm, computer code is written in $C^{++}$ language.

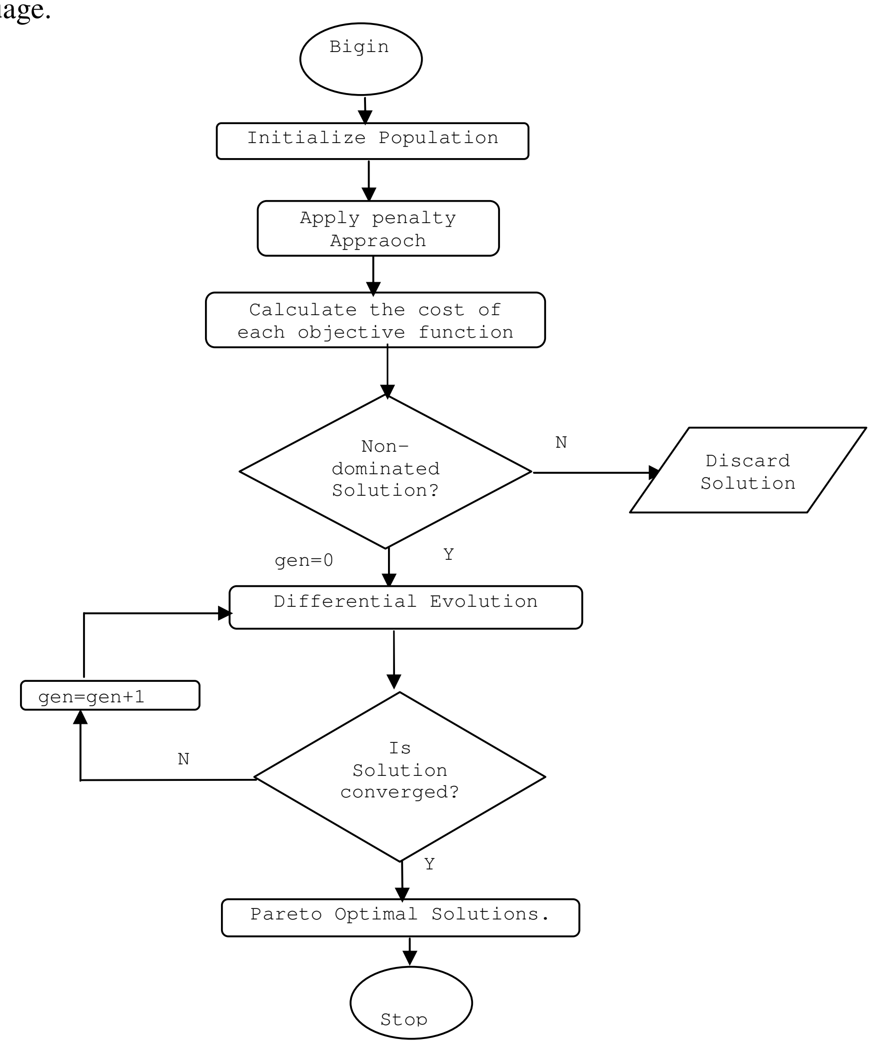


**Figure 1: MODE Schematic Flow Chart.**

## PROBLEM FORMULATION:

Five well-known multi-objective optimization problems are considered for this study. All the problems are having two objectives with/without constraints and with two variables. These problems are formulated as below.

**Test Problem -1** (Deb, 2001) :

$$\begin{aligned} &Minimize \;\; f_1(d,l) = \rho \frac{\pi d^2 l}{4} \\ &Minimize \;\; f2(d,l) = \delta = \frac{64Pl^3}{3E\pi d^4} \end{aligned} \qquad (1)$$

*Subject to* $\sigma_{\max} \le S_y, \quad \delta \le \delta_{\max};$

$$\sigma_{\max} = \frac{32Pl}{\pi d^3}; \quad 10 \le d \le 50\,mm, \quad 200 \le l \le 1000\,mm$$

$\rho$= 7800 kg/m$^3$, $P$= 1 kN, $E$=207 Gpa, $S_y$=300 Mpa, $\delta_{max}$= 5 mm

**Test Problem -2** (Deb, 2001)**:**

$$\begin{aligned} &\textit{Minimize} \quad f_1(x) = x_1 \\ &\textit{Minimize} \quad f_2(x) = \frac{1+x_2}{x_1} \\ &\textit{Subject to} \quad 0.1 \le x_1 \le 1 \\ &\qquad\qquad\quad 0 \le x_2 \le 5 \end{aligned} \tag{2}$$

**Test Problem -3** (Deb, 2001)**:**

$$\begin{aligned} &\textit{Maximize} \quad f_1(x) = 1.1 - x_1 \\ &\textit{Maximize} \quad f_2(x) = 60 - \frac{1+x_2}{x_1} \\ &\textit{Subject to} \quad 0.1 \le x_1 \le 1 \\ &\qquad\qquad\quad 0 \le x_2 \le 5 \end{aligned} \tag{3}$$

**Test Problem -4** (Belegundu and Chandragupta, 2002)**:**

$$\begin{aligned} &\textit{Maximize} \quad f_1(x) = 3x_1 + x_2 + 1 \\ &\textit{Maximize} \quad f_2(x) = -x_1 + 2x_2 \\ &\textit{Subject to} \quad 0 \le x_1 \le 3 \\ &\qquad\qquad\quad 0 \le x_2 \le 3 \end{aligned} \tag{4}$$

**Test Problem -5** (Changkong and Haimes, 1983) :

$$\begin{aligned} &\textit{Minimize} \quad f_1(x) = 2 + (x_1 - 2)^2 + (x_2 - 1)^2, \\ &\textit{Minimize} \quad f_2(x) = 9x_1 - (x_2 - 1)^2, \\ &\textit{Subject to} \quad C_1(x) = x_1^2 + x_2^2 \le 225, \\ &\qquad\qquad\quad C_2(x) = x_1 - 3x_2 + 10 \le 0, \\ &\qquad\qquad\quad -20 \le x_1 \le 20, \\ &\qquad\qquad\quad -20 \le x_2 \le 20. \end{aligned} \tag{5}$$

**SIMULATION RESULTS AND DISCUSSION:**

Above-mentioned Test Problems are simulated using a MODE algorithm. The code is written in C$^{++}$ language. Various key parameters used in the simulation results are listed in Table 1. The result of Test Problems is discussed below.

**Test Problem -1:** Application of MODE is tested on an engineering problem. This is a cantilever design problem with two decision variables, i.e., diameter *(d)* and length *(l)*. In this problem two conflicting objectives are minimization of the weight, $f_1$ and minimization of end deflection, $f_2$. The first objective will resort to an optimum solution having the smaller dimensions of *d* and *l*, so that the overall weight of the beam is minimum. Since the dimensions are small, the beam will not be adequately rigid and the end deflection of the beam will be large. On the other hand, if the beam is minimized for end deflection, the dimensions of the beam are expected to be large, thereby making the weight of the beam large. This problem contains two constraints. The feasible decision variable space for this problem is shown in Fig. 2. The decision variable space is lying in the bound of the variables, which are given the Test Problem -1. Objective space for this problem is shown in Fig. 3 and 4. Fig. 3 shows the feasible objective space. As can be seen from the figure for the weight in the range of 2-3 kg, the deflection in the cantilever can be as high as 3000 mm. Because of huge range of both variables the feasible objective space is very large. The magnified view of objective space along with the Pareto front is shown in Fig. 4.

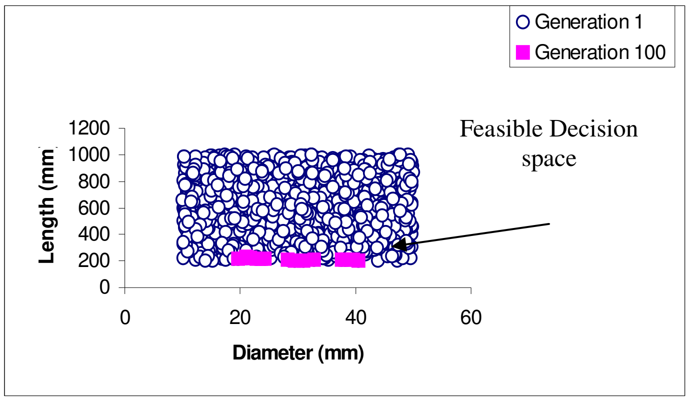


**Figure 2 : Decision variable Space for Test Problem – 1 (Cantilever design).**

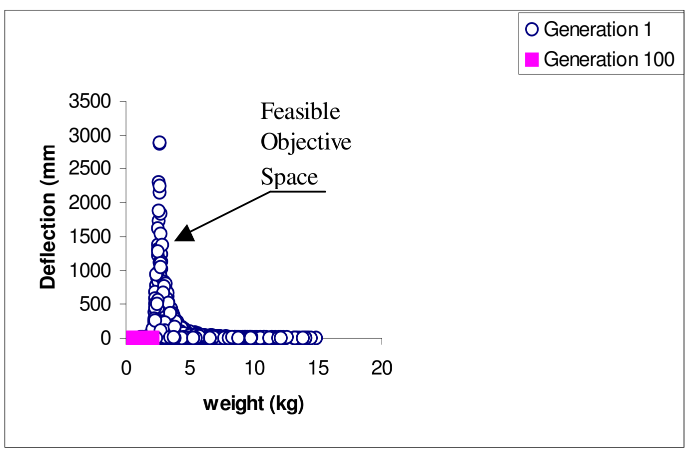


**Figure 3 : Feasible Objective Space for Test Problem -1 (Cantilever design).**

**Test Problem -2:** Although this problem looks simple, it produces conflicting scenarios between both the objectives. Second objective has first objective in its denominator. It is interesting to note that though it is a *min-min* type of Multi-objective optimization problem, the pareto front obtained does not correspond to lower left portion of the objective space. Because $f_1$ appears in the denominator of $f_2$ (which means though $f_1$ has to be minimized independently as far as $f_2$ is concerned, $f_1$ has to be maximized). This is clearly evident from Fig. 6, as the Pareto optimal front lies at the bottom portion of the objective space extending towards right. Fig. 5 shows the decision variable space along with the Pareto decision variables at 100$^{th}$ Generation. The decision variables are found to lie well in the limit of the decision variables as given in Test Problem -2. The Pareto decision variables are found to lie within $x_1$ values of 0.1 to 0.8 (maximum) with very low value of $x_2$ (minimum) as shown in Fig. 5. Fig. 6 shows the objective space and Pareto front for Test Problem -2. The objective space for Test Problem -2 is found to be dense at low values of function $f_2$. This is in agreement with the discussion made above.

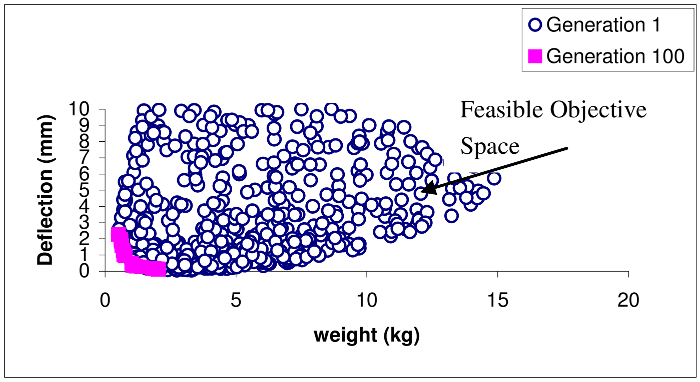


**Figure 4 : Magnified view of Objective Space for Test Problem -1 (Cantilever design).**

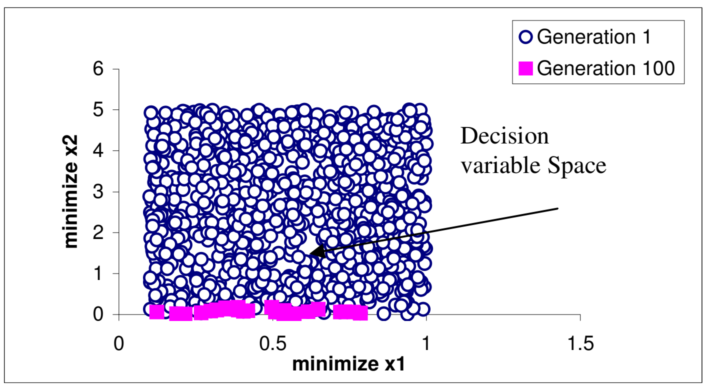


**Figure 5 : Decision variable space for Test Problem -2.**

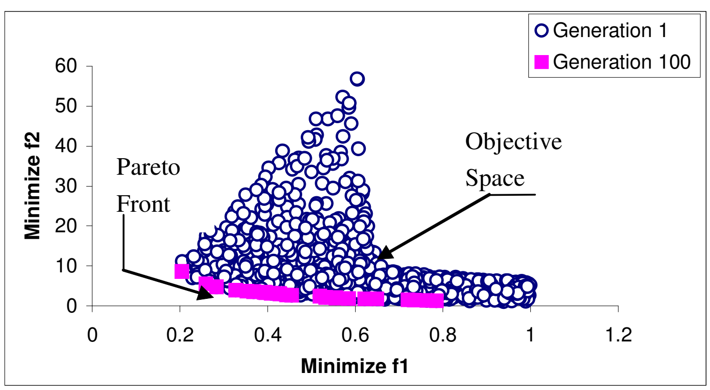


**Figure 6 : Objective Space and Pareto Front for Test Problem -2.**

**Test Problem -3:** Fig. 7 shows the decision variable space for this problem. It can be noted that Pareto decision variables are at $x_2 = 0$ and $x_1 \in [0.17, 0.72]$ except one point $x_2$=2.34. This is due to the nonlinear nature of objective function 2, with respect to objective function 1. The objective space for the *max-max* problem is shown in Fig. 8. Both upper and lower bounds of the objective space are convex in nature, which has resulted in a convex Pareto optimal front.

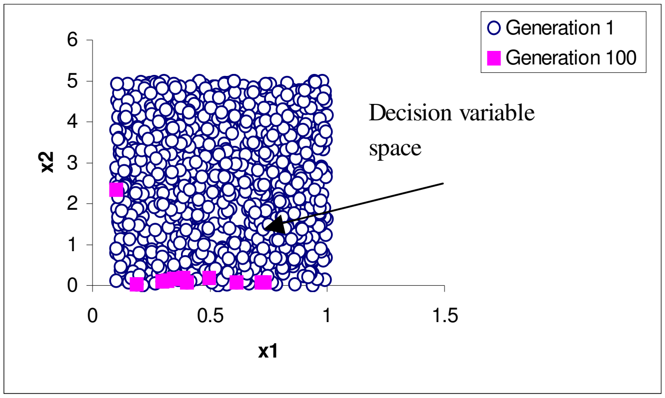


**Figure 7 : Decision variable space for Test Problem -3**

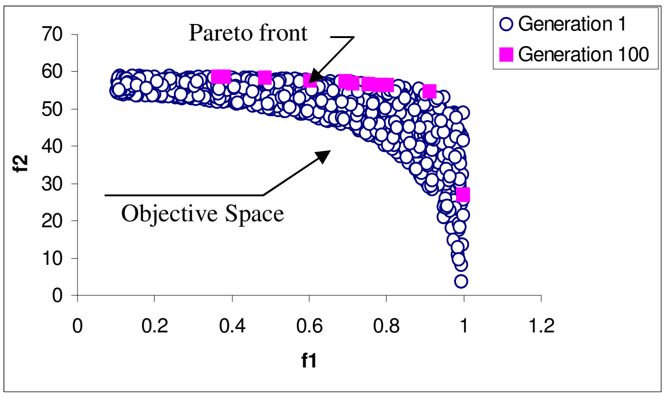


**Figure 8 : Objective Space and pareto front for Test Problem -3.**

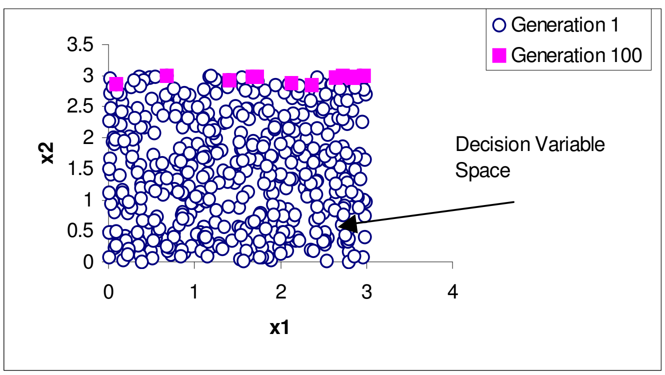


**Figure 9 : Feasible decision variable space for Test Problem -4.**

**Test Problem -4:** The decision variable space for Test Problem -4 is shown is shown in Fig. 9. The decision variables are found to be evenly spreaded across the decision space. The Pareto decision space is found to lie in the upper region of the decision space. Fig. 10 shows the feasible objective space and the Pareto front for the Test Problem -4. It is interesting to see the nature of the objective space versus the nature of decision space. The decision space is rectangular in shape, while the objective space is in the shape of a parallelogram. The

Pareto decision variables lie in the upper limit of rectangle while the Pareto front lie in the upper limit of the objective space. The tilt in the objective space is basically due to the objective function 2, which contains a negative value of $x_1$ variable.

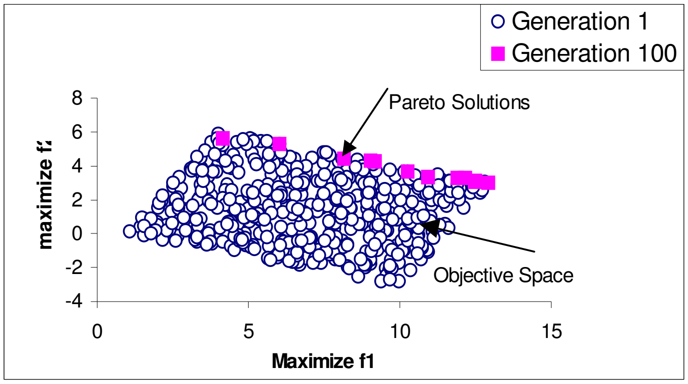


**Figure 10 : Objective Space for Test Problem -4.**

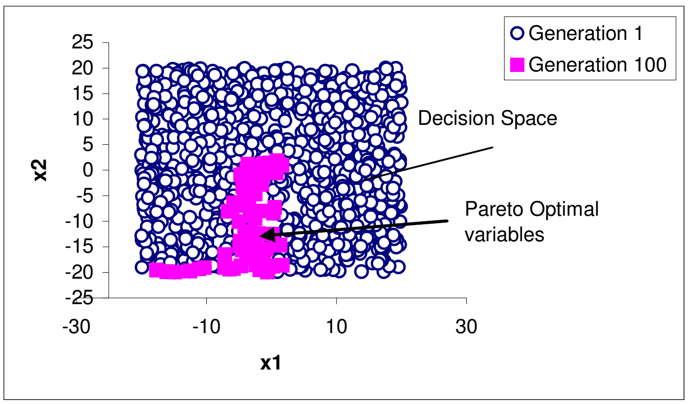


**Figure 11 Constrained Decision Variable Space for Test Problem -5.**

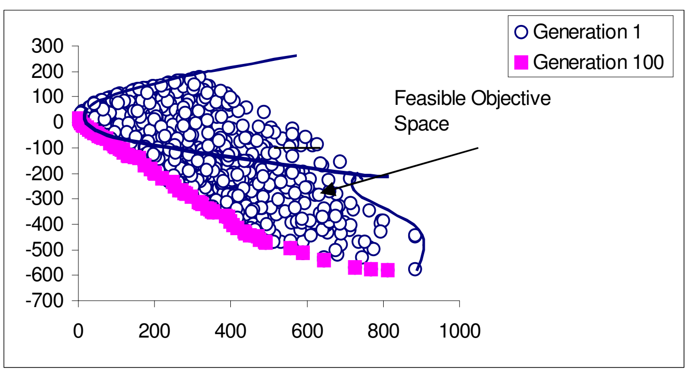


**Figure 12: Feasible Objective Space for Test Problem -5 .**

**Test Problem -5:** The constrained decision variable space for the Test Problem -5 is shown in Fig. 11. The decision space is found to be widely spreaded within the range of variables i.e. $-20 \le x_1, x_2 \le 20$. But the Pareto optimal set for decision variables is found to be in the range of $x_1 \in [-18, 1.65]$ and $x_2 \in [-18, 1.8]$. The feasible objective space along with the Pareto optimal solutions is shown in Fig. 12. The solid line marks the boundary of fold in the objective space. This test problem contains two constraints (one non-linear) and hence the constrained Pareto Optimal front is the shape of as shown in Fig. 12. A careful analysis of the constraints and the objective functions reveals the constrained Pareto Optimal front, as shown in Fig. 12.

**CONCLUSIONS:**

MODE algorithm is applied successfully on the above-mentioned test problems. Simulation runs are performed with various MODE parameters. The parameter values used in MODE simulations for different test problems are shown in Table 1. A uniformly distributed population for the decision variables is found for all Test Problems. The variations in the decision variable space and objective function test and its effect on the Pareto optimal front are discussed. The objective space and decision variable space and the Pareto optimal front is reported for all Test Problems.

**NOMENCLATURE:**

CR Crossover Constant.
F Scaling Factor.
Ng Number of Generations.
NP Number of Population points.
R Penalty Parameter.

**Table 1**: Parameter values used in MODE simulations for different test problems

| Test Problem | NP | CR | Ng | F | R |
|---|---|---|---|---|---|
| 1 | 1000 | 0.9 | 100 | Random | 1.0 |
| 2 | 1000 | 0.9 | 100 | Random | -- |
| 3 | 1000 | 0.9 | 100 | Random | -- |
| 4 | 1000 | 0.9 | 100 | Random | -- |
| 5 | 1000 | 0.9 | 100 | Random | 0.01 |

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