Optimal Design of Gas Transmission Network Using Differential Evolution

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Abstract

Differential Evolution (DE), an evolutionary computation technique, is applied for the optimal design of gas transmission network in this study. The design of an efficient and economical network involves many parameters in a gas transmission system such as source of gas, delivery sites with pipeline segments and compressors, etc. In addition, there are many equality and inequality constraints to be satisfied making the problem highly complex. Hence an efficient strategy is needed in searching for the global optimum. DE has been successfully applied for this complex and highly non-linear problem. The results obtained are compared with those of nonlinear programming technique and branch and bound algorithm. DE is able to find an optimal solution satisfying all the constraints. The proposed strategy takes less computational time to converge when compared to the existing techniques without compromising with the accuracy of the parameter estimates.

1. Introduction

The optimization of non-linear constrained problems is relevant to chemical engineering practice [1, 2]. In recent years, evolutionary algorithms (EAs) have been applied to the solution of NLP in many engineering applications. The best-known algorithms in this class include Genetic Algorithms (GA), Evolutionary Programming (EP), Evolution Strategies (ES) and Genetic Programming (GP). There are many hybrid systems, which incorporate various features of the above paradigms and consequently are hard to classify, which can be referred just as EC methods [3]. They differ from the conventional algorithms since, in general, only the information regarding the objective function is required. EC methods have been applied to a broad range of activities in process system engineering including modeling, optimization and control. Onwubolu and Babu [4] presented various engineering applications using different evolutionary algorithms.

Differential Evolution (DE), developed by [5], is one of the best EC methods. This method provides one of the best genetic algorithms for solving the real-valued test function. The convergence speed of DE is very high. DE has been successfully applied to many complex problems [6-30].

In this study, DE – an evolutionary computation method, is used to solve the classical problem of gas transmission network.

2. Gas Transmission Network

In this age of high competition in the several industries, it becomes necessary to cut down capital and operating costs as much as possible. Specifically in case of Chemical industries, the main focus is on reducing the processing costs, which include heating, cooling, transfer of various streams involved in any operational unit. The gas transmission network, which forms a considerable fraction of the operating cost, is also one of the focus areas. Broadly, a gas transmission system includes source of gas, delivery sites with the pipeline segments and compressor stations used to achieve desired pressure at the delivery site. As the design of an efficient and economical gas transmission network involves a lot of design parameters which directly/indirectly affect the capital and operating costs, this topic deserves special attention. Over the years, various aspects of the problem have been addressed [31-38]. Larson and Wong [31] determined the steady state optimal operating conditions of a straight natural pipeline with compressors in series using dynamic programming to find the optimal suction and discharge pressures. The length and diameter of the pipeline segment were assumed to be constant because of limitations of dynamic programming. Martch and McCall [33] modified the problem by adding branches to the pipeline segments. However, the transmission network was predetermined because of the limitations of the optimization technique used. Cheesman [36] introduced a computer optimizing code in addition to Martch and McCall [33] problem. They considered the length and diameters of the pipeline segments to be variables. But their problem formulation did not allow unbranched network, so complicated network systems couldn’t be handled. Olorunniwo [39] and Olorunniwo and Jensen [40] provided further breakthrough by optimizing a gas transmission network including the following features:

1. The maximum number of compressor stations that would ever be required during the specified time horizon.
2. The optimal location of these compressor stations.
3. The initial construction dates of the stations.
4. The optimal solutions of expansion for the compressor stations.
5. The optimal diameter sizes of the main pipes for each arc of the network.
6. The minimum recommended thickness of the main pipe.
7. The optimal diameter sizes, thicknesses and lengths of any required parallel pipe loops on each arc of the network.
8. The timing of construction of the parallel pipe loops
9. The operating pressures of the compressors and the gas in the pipelines.

They used dynamic programming coupled with optimization logic to find the shortest route through the network.

Edgar & Himmelblau [41] simplified the problem to make sure that the various factors involved in the design are clear. They assumed the gas quantity to be transferred along with the suction and discharge pressures to be given in the problem statement. They optimized the following variables:

1. The number of compressor stations
2. The length of pipeline segments between the compressors stations
3. The diameters of the pipeline segments
4. The suction and discharge pressures at each station.

They considered the minimization of the total cost of operation per year including the capital cost in their objective function against which the above parameters are to be optimized. Edgar and Himmelblau [41] also considered two possible scenarios:

1. The capital cost of the compressor stations is linear function of the horse power
2. The capital cost of the compressor stations is linear function of the horsepower with a fixed capital outlay for zero horsepower.

The first scenario is easy to solve as compared to the second one. They solved the second scenario using the branch and bound technique.

3. Problem formulation

The pipeline configuration is same as chosen by Edgar and Himmelblau [41]. Each of the compressor stations is represented by a node and each of the pipeline segments by an arc. Pressure is assumed to be increasing at a compressor and decreasing along the pipeline segment. The transmission system is presumed to be horizontal. This is a simple example chosen to illustrate a gas transmission system. However, a much more complicated network can be accommodated including various branches and loops at the cost of additional execution time.

Edgar and Himmelblau [41] distinguished between two related problems (one is of a higher degree of difficulty than the other) before proceeding ahead with the details of the design problem. If the capital costs of the compressors are linear functions of horsepower, then the transmission line problem can be solved as a nonlinear programming problem by one of the methods discussed by Edgar and Himmelblau [41].

Alternately, if the capital costs are a linear function of horsepower with a fixed capital outlay for zero horsepower, a condition that is more closely represents the practical problems, then the design problem becomes more difficult to solve and a branch-and-bound algorithm combined with a nonlinear programming algorithm has to be used.

3.1 Number of Variables

Each node and each arc are labeled separately for a given pipeline configuration. The number of variables is as following:

• Total Compressors	N
• Suction Pressures	N-1
• Discharge Pressures	N
• Pipeline Lengths & Diameters	N+1

3.2 Variables

Each pipeline segments has the following variables associated with it:

1. Flow rate
2. Initial pressure
3. Outlet pressure
4. Pipe diameter
5. Pipeline segment length

It is assumed that each of the compressors has gas losses of one-half of one percent of the gas transmitted. As the mass flow rate is fixed, only the last four variables become important and need to be determined for each segment in the present problem.

3.3 Assumptions

The following assumptions are made:

1. Each compressor functions adiabatically with an inlet temperature equal to that of the surroundings.
2. Pipeline segment is long so that by the time gas reaches the next compressor it returns to the ambient temperature.
3. The annualized capital costs for each pipeline segment depend on pipe diameter and length, and have been taken as \$870/(inch)(mile)(year) as reported by Martch and McCall [33].
4. The rate of work of one compressor is estimated using the following correlation:

   $$W = (0.08531)Q\frac{k}{k-1}T_1\left[\left(\frac{p_d}{p_s}\right)^{z(k-1)/k} - 1\right]$$

   where
   $k = C_p/C_v$ for gas at suction conditions = 1.26 [42]
   z = compressibility factor of gas at suction conditions
   p_s = suction pressure, psia
   p_d = discharge pressure, psia
   T_i = suction temperature = 520^0R
   Q = flow rate into the compressor, MMCFD (million cubic feet per day)
   W = rate of work, horsepower
5. Total operating costs are linear function of compressor horsepower (Operation and maintenance costs per year can be related directly to horsepower [37] and have been estimated to be between 8.00 and 14.0 \$/(hp)(year) [33].
6. In first scenario, the cost is a linear function of horsepower (\$70.00/(hp)(year)) with the line passing through the origin.
7. In second scenario, the cost is a linear function of horsepower with a fixed initial capital outlay (\$70.00/(hp)(year) + \$10,000), which takes the installation costs, foundation, etc. into account.

3.4 Objective Function

As the objective in this study is to minimize the cost, the objective function comprises of the sum of the yearly operating and maintenance costs of the compressors in addition to the sum of the discounted capital costs of the pipeline segments and compressors over a period of 10 years. For line A, the objective function for the problem chosen in dollars per year is:

$$f = \sum_{i=1}^{n}(C_0 + C_c)Q_i(0.08531)T_1\left(\frac{k}{k-1}\right)\left[\left(\frac{p_{d_i}}{p_{s_i}}\right)^{z(k-1/k)} - 1\right] + \sum_{j=1}^{m} C_s L_j D_j$$

where,

n = number of compressors in the system
m = number of pipeline segments in the system (= n + 1)
C_0 = annual operating cost, \$/(hp)(year)
C_c = compressor capital cost, \$/(hp)(year)
C_s = pipe capital cost, \$/(in)(mile)(year)
L_j = length of pipeline segment j, mile
D_j = diameter of pipeline segment j, inch

Edgar and Himmelblau [41] justified the reason why a branch and bound technique is required to solve the design problem for second scenario along with non-linear programming because of the limitations of non-linear programming. However, DE has the capability of dealing with above complications as it is a population-based search algorithm, and hence, it is used in this study for both the possible scenarios.

3.5 Inequality Constraints

A constraint is there for operation of each compressor as the discharge pressure is always greater than or equal to the suction pressure:

$$\frac{p_{d_i}}{p_{s_i}} \geq 1 \qquad i = 1, 2, \ldots\ldots, n$$

and the compressor ratio should not exceed some prespecified maximum limit K

$$\frac{p_{d_i}}{p_{s_i}} \leq K_i \qquad i = 1, 2, \ldots\ldots, n$$

Also, the upper and lower bounds are placed on each of the four variables

$$p_{d_i}^{\min} \leq p_{d_i} \leq p_{d_i}^{\max}$$

$$p_{s_i}^{\min} \leq p_{s_i} \leq p_{s_i}^{\max}$$

$$L_i^{\min} \leq L_i \leq L_i^{\max}$$
$$D_i^{\min} \leq D_i \leq D_i^{\max}$$

3.6 Equality Constraints

For the gas transmission network chosen in the problem, there are two classes of equality constraints. First, as the length of the system is fixed, there would be two constraints for two branches as given below:

$$\sum_{j=1}^{N1-1} L_j + \sum_{j=N1}^{N1+N2} L_j = L_1^*$$

$$\sum_{j=1}^{N1-1} L_j + \sum_{j=N1+N2+1}^{N1+N2+N3+1} L_j = L_2^*$$

where L_k^* represents the length of a branch. Secondly, each pipeline segment must satisfy the Weymouth flow equation [43]:

$$Q_j = 871 D_j^{8/3}\left[\frac{p_d^2 - p_s^2}{L_j}\right]^{1/2}$$

where Q_j is a fixed number, and p_d & p_s are the discharge pressure & suction pressure at the entrance and exit of the segment respectively.

4 Results & Discussion

The seventh of the ten DE strategies (DE/rand/1/bin) proposed by Price & Storn [44] is used for solving this problem. The key parameters of DE are: NP, the population size; CR, the crossover constant; and F, scaling factor.

If the objective function is formulated in terms of cost, the vector that yields the lesser cost replaces the population member in the initial population. If the objective function is in terms of profit function, then the vector with greater profit replaces the population member in the initial population. This procedure is continued till some stopping criterion is met. This may be of two kinds. One may be some convergence criterion that states that the error in the minimum or maximum between two previous generations should be less than some specified value (standard deviation may be used). The other may be an upper bound on the number of generations. The stopping criteria may be a combination of the two as well. Either way, once the stopping criterion is met, the computations are terminated.

Fig. 1 shows the design problem outlined. The maximum number of compressors in branches 1, 2, and 3 are set at 4, 3, and 3 respectively. The input pressure was fixed at 500 psia at a flow rate of 600 MMCFD, and the two output pressures are 600 and 300 psia respectively for branches 2 and 3. The total length of the branches 1 and 2 put together is constrained to be 175 miles, whereas the total length of the branches 1 and 3 put together is constrained to be 200 miles.

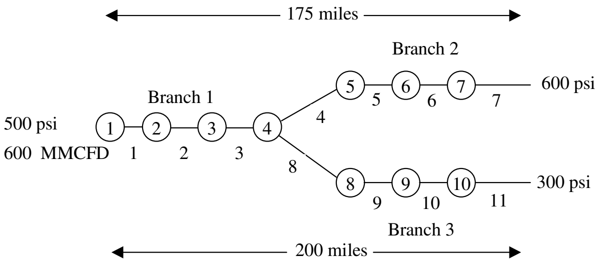

Fig. 1. Initial configuration of gas transmission network.

The upper bound for diameter of pipeline segments in branch 1 is set at 36 inches, and those for branches 2 and 3 are set at 18 inches. The lower bound on the diameter of all pipeline segments is set at 4 inches. A lower bound of 2 miles is placed on each pipeline segment to ensure that the natural gas is at ambient conditions when it entered at subsequent compressor in the pipeline.

The resulting solution obtained by Edgar and Himmelblau [41] to the above problem using the cost relation of first scenario (using non-linear programming) and for second scenario (using branch and bound technique) are shown in Fig. 2 and Fig. 3 respectively. The optimum value of objective function for first scenario was reported to be 7.289x10^6 \$/yr. Based on the results listed the calculated optimum value of objective function for second scenario is found to be 7.389x10^6 \$/yr.

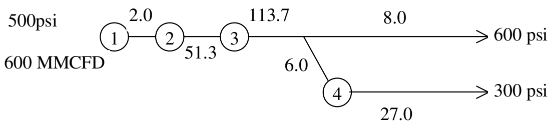

Fig. 2. Final optimal gas transmission network for first possible scenario [41]

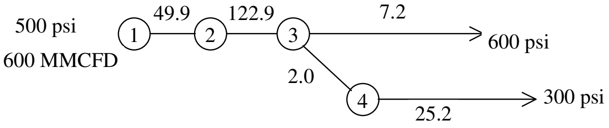

Fig. 3. Final optimal gas transmission network for second possible scenario [41]

The discharge pressure of pipeline segment 9 reported by them does not satisfy the second equality constraint. The value of the compression ratio of third compressor station for first scenario does not match with the corresponding ratio of discharge and suction pressures. Edgar and Himmelblau [41] modified the second equality constraint to avoid problems in taking square roots by squaring it as given below:

$$(871)^2 D_j^{16/3}\left(p_d^2 - p_s^2\right) - L_j Q_j^2 = 0$$

But, none of their results satisfy the above constraint.

All problems mentioned above have been addressed and sorted out in this study using DE. The resulting solution obtained to the design problem as shown in Fig. 1 using the cost relation of first & second scenarios are shown in Fig. 4 & Table-1.

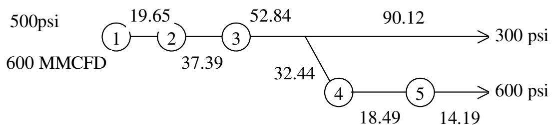

Fig. 4. Final optimal gas transmission network for both the scenarios (Present study)

Table-1. Values of operating variables for the optimal network configuration using both the scenarios.

L_i	p_d (psia)	p_s (psia)	D_i (inch)	Length (mile)	Q_i (MMCFD)
1	730.56	691.44	34.77	19.65	597.000
2	942.47	852.83	32.08	37.39	594.015
3	865.68	736.92	32.61	52.84	591.045
4	736.92	690.05	24.33	14.51	294.022
5	806.07	715.99	22.09	17.93	292.552
6	715.99	620.00	22.45	18.49	291.079
7	663.72	600.00	23.27	14.19	289.624
8	970.15	775.08	21.03	33.88	294.022
9	775.08	749.88	22.76	5.89	292.552
10	749.88	711.46	21.40	6.27	291.079
11	711.46	300.00	21.15	44.07	289.624

Compressor station	Compression ratio	Capital cost (\$/year)
1	1.4611	70.00
2	1.3630	70.00
3	1.0150	70.00
4	1.0000	70.00
5	1.1681	70.00
6	1.0000	70.00
7	1.1062	70.00
8	1.0000	70.00
9	1.0000	70.00
10	1.0000	70.00

As can be seen from the above results, all the constraints are satisfied with this optimal gas transmission network. Also, a single network has been obtained using DE for both the possible scenarios described earlier. The optimum values of objective function for both the possible scenarios are obtained as $7.692\text{x}10^{6}$ \$/yr and $7.792\text{x}10^{6}$ \$/yr respectively. Though the objective function values are slightly higher than those reported by Edgar and Himmelblau [41], these values correspond to satisfying all the constraints. In addition, by approximating the compression ratio of compressor station 3 (1.0150 in Table-1) to 1.00 (considering only two digits as Edgar and Himmelblau, [41]), it is possible to remove one more compressor from the proposed final network (Fig. 4), in which case the total number of compressors would be only 4 and the final objective function value would further reduce.

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