Voronoi Diagram

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Voronoi diagram is a very useful tool in Computational Geometry. Apart from many other problems, its main application is in the geometric proximity queries. The origin of this topic is found in the book titled "Principia Philosophiae" by R. Descartes in the year 1644. There he mentioned that the solar system can be partitioned into many convex regions. Each region corresponds to a *star*. The matters in each region are revolving around its corresponding star.

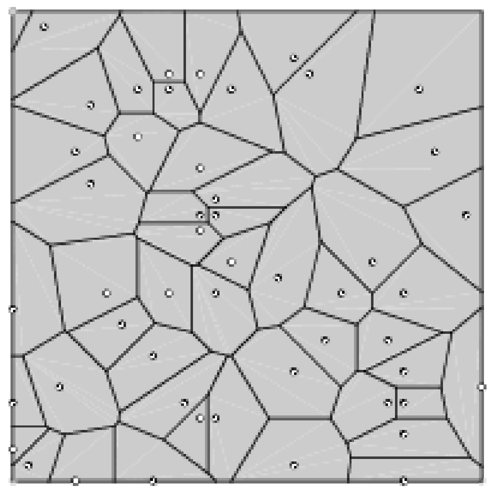

Figure 1: Voronoi diagram of a point set in 2D

In the recent days, we find the mention of *Post Office Problem* in the book titled "The Art of Computer Programming - III: Sorting and Searching" by Professor Donald E. Knuth in the year 1973. The problem is stated as follows: Let P be a set of points distributed on a 2D plane, which correspond to post offices in a city. The objective is to partition the plane (city) into regions such that each region corresponds to one post office in P, and for any arbitrary point q on the plane, its nearest post office is the member $p \in P$ whose region contains q (see Figure 1). The Voronoi region of a point $p \in P$ is obtained as follows: For each point $q \in P$, draw the perpendicular bisector of p and q, and consider the half-plane containing p. The convex region, obtained by the intersection of these half-planes, define the Voronoi region of the point p. The Voronoi regions of all the members in P is the subdivision of the entire region into a set of convex regions, each correspond to a point in P. This geometric structure is referred to as Voronoi diagram $\mathcal{V}(P)$ of the point set P.

In general, the Voronoi diagram may be defined for a set of arbitrary geometric objects P in any dimensional plane.

We shall illustrate different important properties of the Voronoi diagram of a point set P, construction of the Voronoi diagram $\mathcal{V}(P)$, and some important applications of $\mathcal{V}(P)$. Next, we will define the furthest point Voronoi diagram and higher order Voronoi diagram. The furthest point Voronoi diagram is a partition of the region into convex pieces such that the furthest member of P for every point in a particular region is the same. Similarly, order k Voronoi diagram is also a partition of the entire region into convex pieces such that for each point in a particular region, its k nearest neighbors in P are the same. Several other variations of the Voronoi diagram and their uses will be discussed.